Logic

- Aristotle
  (384–322 BC)
- Theophrastus
  (371 – c. 287 BC)

What is Logic?

- A process of reasoning is logical if it always arrives at correct conclusions given correct information.
- Illogical reasoning can come to any conclusion given the facts.

Motivation

- A general framework to express and solve problems.

Types of Logical Reasoning

- Deduction
- Induction
- Abduction

Modus Ponens

$$\text{premise} \implies \text{conclusion}$$

Propositional Logic

- Variables - True/False
- Sentences - Rules
- Models - Assignments of variables to satisfy the rules

Operators

- $\wedge$ - And
- $\vee$ - Or
- $\neg$ - Not
- $x \implies y$ - Implication (Conditional)
- $x \iff y$ - Bidirectional Implication (Conditional)

Rock-Paper-Scissors

- Variables for player moves
  $P_{1,R}, P_{1,S}, P_{1,P}, P_{2,R}, P_{2,S}, P_{2,P}$
- Variables for wins and draw
  W_1, W_2, D
- Rules for win
  $$\begin{aligned} P_{1,R} \wedge P_{2,S} &\implies W_1 \\ P_{1,R} \wedge P_{2,P} &\implies W_2 \\ P_{1,S} \wedge P_{2,R} &\implies W_2 \\ P_{1,S} \wedge P_{2,P} &\implies W_1 \\ P_{1,P} \wedge P_{2,R} &\implies W_1 \\ P_{1,P} \wedge P_{2,S} &\implies W_2 \end{aligned}$$
- Rules for draw
  $$\begin{aligned} P_{1,R} \wedge P_{1,R} &\implies D \\ P_{1,P} \wedge P_{1,P} &\implies D \\ P_{1,S} \wedge P_{1,S} &\implies D \end{aligned}$$
- Rules for exclusive play
  $$\begin{aligned} P_{1,R} &\implies \neg P_{1,S} \wedge \neg P_{1,P} \\ P_{1,P} &\implies \neg P_{1,S} \wedge \neg P_{1,R} \\ P_{1,S} &\implies \neg P_{1,P} \wedge \neg P_{1,R} \\ P_{2,R} &\implies \neg P_{2,S} \wedge \neg P_{2,P} \\ P_{2,P} &\implies \neg P_{2,S} \wedge \neg P_{2,R} \\ P_{2,S} &\implies \neg P_{2,P} \wedge \neg P_{2,R} \end{aligned}$$
- Rules for exclusive win
  $$\begin{aligned} W_1 &\implies \neg W_2 \\ W_2 &\implies \neg W_1 \\ W_1 &\implies \neg D \\ W_2 &\implies \neg D \\ D &\implies \neg W_1 \\ D &\implies \neg W_2 \end{aligned}$$

Satisfiability

- Satisfiability: Is there a model (assignment of variables) that satisfy the sentences (rules)?
- $P \implies Q$

Rock-Paper-Scissors Example

- Variables for player moves
  $P_{1,R}, P_{1,S}, P_{1,P}, P_{2,R}, P_{2,S}, P_{2,P}$
- Variables for wins and draw
  W_1, W_2, D
- Rules for win
  $$\begin{aligned} P_{1,R} \wedge P_{2,S} &\implies W_1 \\ P_{1,R} \wedge P_{2,P} &\implies W_2 \\ P_{1,S} \wedge P_{2,R} &\implies W_2 \\ P_{1,S} \wedge P_{2,P} &\implies W_1 \\ P_{1,P} \wedge P_{2,R} &\implies W_1 \\ P_{1,P} \wedge P_{2,S} &\implies W_2 \end{aligned}$$
- Rules for draw
  $$\begin{aligned} P_{1,R} \wedge P_{1,R} &\implies D \\ P_{1,P} \wedge P_{1,P} &\implies D \\ P_{1,S} \wedge P_{1,S} &\implies D \end{aligned}$$
- Rules for exclusive play
  $$\begin{aligned} P_{1,R} &\implies \neg P_{1,S} \wedge \neg P_{1,P} \\ P_{1,P} &\implies \neg P_{1,S} \wedge \neg P_{1,R} \\ P_{1,S} &\implies \neg P_{1,P} \wedge \neg P_{1,R} \\ P_{2,R} &\implies \neg P_{2,S} \wedge \neg P_{2,P} \\ P_{2,P} &\implies \neg P_{2,S} \wedge \neg P_{2,R} \\ P_{2,S} &\implies \neg P_{2,P} \wedge \neg P_{2,R} \end{aligned}$$
- Rules for exclusive win
  $$\begin{aligned} W_1 &\implies \neg W_2 \\ W_2 &\implies \neg W_1 \\ W_1 &\implies \neg D \\ W_2 &\implies \neg D \\ D &\implies \neg W_1 \\ D &\implies \neg W_2 \end{aligned}$$
- Given facts:
  $P_{1,P} = \text{True},\ P_{2,R} = \text{True}$

Rock-Paper-Scissors Example 2

- Variables for player moves
  $P_{1,R}, P_{1,S}, P_{1,P}, P_{2,R}, P_{2,S}, P_{2,P}$
- Variables for wins and draw
  W_1, W_2, D
- Rules for win
  $$\begin{aligned} P_{1,R} \wedge P_{2,S} &\implies W_1 \\ P_{1,R} \wedge P_{2,P} &\implies W_2 \\ P_{1,S} \wedge P_{2,R} &\implies W_2 \\ P_{1,S} \wedge P_{2,P} &\implies W_1 \\ P_{1,P} \wedge P_{2,R} &\implies W_1 \\ P_{1,P} \wedge P_{2,S} &\implies W_2 \end{aligned}$$
- Rules for draw
  $$\begin{aligned} P_{1,R} \wedge P_{1,R} &\implies D \\ P_{1,P} \wedge P_{1,P} &\implies D \\ P_{1,S} \wedge P_{1,S} &\implies D \end{aligned}$$
- Rules for exclusive play
  $$\begin{aligned} P_{1,R} &\implies \neg P_{1,S} \wedge \neg P_{1,P} \\ P_{1,P} &\implies \neg P_{1,S} \wedge \neg P_{1,R} \\ P_{1,S} &\implies \neg P_{1,P} \wedge \neg P_{1,R} \\ P_{2,R} &\implies \neg P_{2,S} \wedge \neg P_{2,P} \\ P_{2,P} &\implies \neg P_{2,S} \wedge \neg P_{2,R} \\ P_{2,S} &\implies \neg P_{2,P} \wedge \neg P_{2,R} \end{aligned}$$
- Rules for exclusive win
  $$\begin{aligned} W_1 &\implies \neg W_2 \\ W_2 &\implies \neg W_1 \\ W_1 &\implies \neg D \\ W_2 &\implies \neg D \\ D &\implies \neg W_1 \\ D &\implies \neg W_2 \end{aligned}$$
- Given facts:
  $P_{1,P} = \text{True}, \ W_1 = \text{True}$
  $P_{2,P} = \text{True}$

Rock-Paper-Scissors Example 3

- Variables for player moves
  $P_{1,R}, P_{1,S}, P_{1,P}, P_{2,R}, P_{2,S}, P_{2,P}$
- Variables for wins and draw
  W_1, W_2, D
- Rules for win
  $$\begin{aligned} P_{1,R} \wedge P_{2,S} &\implies W_1 \\ P_{1,R} \wedge P_{2,P} &\implies W_2 \\ P_{1,S} \wedge P_{2,R} &\implies W_2 \\ P_{1,S} \wedge P_{2,P} &\implies W_1 \\ P_{1,P} \wedge P_{2,R} &\implies W_1 \\ P_{1,P} \wedge P_{2,S} &\implies W_2 \end{aligned}$$
- Rules for draw
  $$\begin{aligned} P_{1,R} \wedge P_{1,R} &\implies D \\ P_{1,P} \wedge P_{1,P} &\implies D \\ P_{1,S} \wedge P_{1,S} &\implies D \end{aligned}$$
- Rules for exclusive play
  $$\begin{aligned} P_{1,R} &\implies \neg P_{1,S} \wedge \neg P_{1,P} \\ P_{1,P} &\implies \neg P_{1,S} \wedge \neg P_{1,R} \\ P_{1,S} &\implies \neg P_{1,P} \wedge \neg P_{1,R} \\ P_{2,R} &\implies \neg P_{2,S} \wedge \neg P_{2,P} \\ P_{2,P} &\implies \neg P_{2,S} \wedge \neg P_{2,R} \\ P_{2,S} &\implies \neg P_{2,P} \wedge \neg P_{2,R} \end{aligned}$$
- Rules for exclusive win
  $$\begin{aligned} W_1 &\implies \neg W_2 \\ W_2 &\implies \neg W_1 \\ W_1 &\implies \neg D \\ W_2 &\implies \neg D \\ D &\implies \neg W_1 \\ D &\implies \neg W_2 \end{aligned}$$
- Given facts:
  $P_{1,S} = \text{True},\ W_1 = \text{True}$